Problem 28

\[ p(x) = \frac{1}{5} \]

\[ p(\text{ineffective vial} \mid x) = 10\% \]

\[ p(\text{ineffective vial} \mid x') = 2\% \]

X: the shipment is from company X

I: 1 out of 30 vials tested is ineffective

\[ p(I \mid x) = C_{30}^{1} \cdot (0.1)^{1} \cdot (0.9)^{29} = 0.141 \]

\[ Y \sim \text{Binomial} \left( n = 30, p = 1 \right) \]

\[ p(Y = 1) \]

\[ Y \sim \text{Binomial} \left( n = 30, p = 0.02 \right) \]

\[ p(I \mid x') = C_{30}^{1} \cdot (0.02)^{1} \cdot (0.98)^{29} = 0.334 \]

\[ p(x \mid I) = \frac{p(I \mid x) \cdot p(x)}{p(I \mid x) \cdot p(x) + p(I \mid x') \cdot p(x')} \]

\[ = \frac{0.141 \cdot \frac{1}{5}}{0.141 \cdot \frac{1}{5} + 0.334 \cdot \frac{1}{5}} = 0.296 \]