56. Solution: C
Let Y represent the payment made to the policyholder for a loss subject to a deductible D.
That is \( Y = \begin{cases} 0 & \text{for } 0 \leq X \leq D \\ X - D & \text{for } D < X \leq 1 \end{cases} \)
Then since \( E[X] = 500 \), we want to choose D so that
\[
\frac{1}{4} \int_0^{1000} \frac{1}{1000} (x - D) \, dx = \frac{1}{1000} \int_D^{1000} \frac{(x - D)^2}{2} \, dx = \frac{(1000 - D)^2}{2000}
\]
\[(1000 - D)^2 = 2000/4 \cdot 500 = 500^2
1000 - D = 500
D = 500 \text{ (or } D = 1500 \text{ which is extraneous).}

57. Solution: B
We are given that \( M_x(t) = \frac{1}{(1 - 2500r)^t} \) for the claim size X in a certain class of accidents.
First, compute \( M'_x(t) = \frac{(-4)(-2500)}{(1 - 2500r)^2} = \frac{10,000}{(1 - 2500r)^2} \)
\( M''_x(t) = \frac{(10,000)(-5)(-2500)}{(1 - 2500r)^3} = \frac{125,000,000}{(1 - 2500r)^3} \)
Then \( E[X] = M'_x(0) = 10,000 \)
\( E[X^2] = M''_x(0) = 125,000,000 \)
\( \text{Var}[X] = E[X^2] - (E[X])^2 = 125,000,000 - (10,000)^2 = 25,000,000 \)
\( \sqrt{\text{Var}[X]} = 5,000 \).

58. Solution: E
Let \( X_J, X_K, \text{ and } X_L \) represent annual losses for cities J, K, and L, respectively. Then
\( X = X_J + X_K + X_L \) and due to independence
\[
M(t) = E[e^{tX}] = E[e^{t(x_J + x_K + x_L)}] = E[e^{tx_J}]E[e^{tx_K}]E[e^{tx_L}]
= M(t) M_K(t) M_L(t) = (1 - 2t)^{-3} (1 - 2t)^{-2.5} (1 - 2t)^{-4.5} = (1 - 2t)^{-10}
\]
Therefore,
\( M'(t) = 20(1 - 2t)^{-11} \)
\( M''(t) = 440(1 - 2t)^{-12} \)
\( M'''(t) = 10,560(1 - 2t)^{-13} \)
\( E[X^3] = M'''(0) = 10,560 \)