64. Solution: A
Let $X$ denote claim size. Then $E[X] = [20(0.15) + 30(0.10) + 40(0.05) + 50(0.20) + 60(0.10) + 70(0.10) + 80(0.30)] = (3 + 3 + 2 + 10 + 6 + 7 + 24) = 55$
$E[X^2] = 400(0.15) + 900(0.10) + 1600(0.05) + 2500(0.20) + 3600(0.10) + 4900(0.10) + 6400(0.30) = 60 + 90 + 80 + 500 + 360 + 490 + 1920 = 3500$
$\text{Var}[X] = E[X^2] – (E[X])^2 = 3500 – 3025 = 475$ and \( \sqrt{\text{Var}[X]} = 21.79 \).

Now the range of claims within one standard deviation of the mean is given by $[55.00 – 21.79, 55.00 + 21.79] = [33.21, 76.79]$
Therefore, the proportion of claims within one standard deviation is $0.05 + 0.20 + 0.10 + 0.10 = 0.45$.

65. Solution: B
Let $X$ and $Y$ denote repair cost and insurance payment, respectively, in the event the auto is damaged. Then

$$Y = \begin{cases} 0 & \text{if } x \leq 250 \\ x - 250 & \text{if } x > 250 \end{cases}$$

and

$$E[Y] = \int_{250}^{1500} \frac{1}{1500} (x - 250) dx = \frac{1}{3000} (x - 250)^2 \bigg|_{250}^{1500} = \frac{1250^2}{3000} = 521$$

$$E[Y^2] = \int_{250}^{1500} \frac{1}{1500} (x - 250)^2 dx = \frac{1}{4500} (x - 250)^3 \bigg|_{250}^{1500} = \frac{1250^3}{4500} = 434,028$$

$$\text{Var}[Y] = E[Y^2] - (E[Y])^2 = 434,028 - (521)^2$$

$$\sqrt{\text{Var}[Y]} = 403$$

66. Solution: E
Let $X_1, X_2, X_3,$ and $X_4$ denote the four independent bids with common distribution function $F$. Then if we define $Y = \max (X_1, X_2, X_3, X_4)$, the distribution function $G$ of $Y$ is given by

$$G(y) = \Pr[Y \leq y]$$

$$= \Pr[(X_1 \leq y) \cap (X_2 \leq y) \cap (X_3 \leq y) \cap (X_4 \leq y)]$$

$$= \Pr[X_1 \leq y] \Pr[X_2 \leq y] \Pr[X_3 \leq y] \Pr[X_4 \leq y]$$

$$= [F(y)]^4$$

$$= \frac{1}{16} (1 + \sin \pi y)^4, \quad \frac{3}{2} \leq y \leq \frac{5}{2}$$

It then follows that the density function $g$ of $Y$ is given by