Estimate $\Theta$ using moments.

1st moment: $E(x)$

2nd moment: $E(x^2)$

Given:

$\sum x_i = 150$ $\quad \sum x_i^2 = 500$

$E(x) = \sum x_i / 10 = 150/10 = 15$

$E(x^2) = \sum x_i^2 / 10 = 500/10 = 50$

Population density function:

$\frac{1}{2}\left[\frac{1}{\Theta}\exp(-\frac{x}{\Theta}) + \frac{1}{\sigma}\exp(-x/\sigma)\right]$

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$E(x) = \frac{1}{2}E(x)_1 + \frac{1}{2}E(x)_2$

$= \frac{1}{2}\Theta + \frac{1}{2}\sigma$

$= \frac{1}{2}(\Theta + \sigma)$

$E(x^2) = \frac{1}{2}E(x^2)_1 + \frac{1}{2}E(x^2)_2$

$= \frac{1}{2}(2\Theta^2) + \frac{1}{2}(2\sigma^2)$

$= \Theta^2 + \sigma^2$

$E(x) = 15 = \frac{1}{2}(\Theta + \sigma)$

$E(x^2) = 50 = \Theta^2 + \sigma^2$

$30 = \Theta + \sigma \quad \sigma = 30 - \Theta$

$50 = \Theta^2 + (30 - \Theta)^2$

$= 2\Theta^2 - 60\Theta + 900$

$\Theta = 10$ or $20$

if $\Theta = 10$, $30 = 10 + \sigma$

$\sigma = 20$

if $\Theta = 20$, $30 = 20 + \sigma$

$\sigma = 10$

$\Theta = 20$