

Determine  $n$ , such that:

$$\hat{F}(n) = .542$$

1. Determine  $\hat{H}(n)$

- a. Determine surrenders
- b. Determine risk sets

2. Set  $\hat{F}(n)$  in terms of  $\hat{H}(n)$

3. Set  $\hat{F}(n) = .542$  and solve for  $n$ .

Study on 100 policies

Year	Surrendered	riskset	$\frac{\text{surrender}}{\text{riskset}}$
1	1	100	$\frac{1}{100}$
2	2	100	$\frac{2}{100}$
3	3	100	$\frac{3}{100}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n$	$n$	100	$\frac{n}{100}$

$$\hat{H}(n) = \sum_{i=1}^n \left( \frac{\text{surrendered in year } i}{\text{riskset in year } i} \right)$$

$$= \frac{1}{100} + \frac{2}{100} + \frac{3}{100} + \dots + \frac{n}{100}$$

$$= \frac{1}{100} (1 + 2 + 3 + \dots + n)$$

Fact:  $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

$$= \frac{1}{100} \left( \frac{n(n+1)}{2} \right)$$

$$= \frac{n(n+1)}{200}$$

$$\hat{H}(n) = \frac{n(n+1)}{200}$$

$$\hat{F}(n) = 1 - \hat{S}(n) \quad \hat{S}(n) = e^{-\hat{H}(n)}$$

$$\hat{F}(n) = 1 - e^{-\hat{H}(n)}$$

$$.542 = 1 - e^{-\frac{n(n+1)}{200}}$$

$$e^{-\frac{n(n+1)}{200}} = .458$$

$$-\frac{n(n+1)}{200} = -.780886$$

$$n^2 + n - 156.1772189$$

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Using Quadratic Formula:

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = -13.007 \text{ or } 12.0$$

$$n = -13 \text{ or } 12$$

$$n = 12$$