

Given: $F(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}$, $x \geq 0$

policy limit: $u = 1000$

Want: $\bar{E}(X \wedge u) = \bar{E}(X \wedge 1000) = ?$

Shortcut: $\bar{E}(X \wedge u) = -\int_{-\infty}^0 \bar{F}(x) dx + \int_0^u S(x) dx.$

$$= 0 + \int_0^u S(x) dx, \quad (\bar{F}(x) = 0, x \leq 0)$$

$$= \int_0^u 1 - F(x) dx$$

$$\bar{E}(X \wedge 1000) = \int_0^{1000} 1 - F(x) dx = \int_0^{1000} 0.8e^{-0.02x} + 0.2e^{-0.001x} dx$$

$$= 0.8 \times \left(-\frac{1}{0.02} e^{-0.02x} \right) + 0.2 \times \left(-\frac{1}{0.001} e^{-0.001x} \right) \Big|_0^{1000}$$

$$= -40 e^{-0.02x} \Big|_0^{1000} - 200 e^{-0.001x} \Big|_0^{1000}$$

$$= -40 \times (e^{-20} - 1) - 200 (e^{-1} - 1) = 166.4$$

Without the shortcut:

$$E(X \wedge u) = \int_{-\infty}^u x f(x) dx + \int_u^{\infty} u f(x) dx$$

$$= \int_0^{1000} x f(x) dx + \int_{1000}^{\infty} 1000 f(x) dx$$

$$= xF(x) \Big|_0^{1000} - \int_0^{1000} F(x) dx + 1000(1 - F(1000))$$

$$= 1000F(1000) - 0 - \int_0^{1000} F(x) dx + 1000 - 1000F(1000)$$

$$= 1000 - \int_0^{1000} F(x) dx = \int_0^{1000} (1 - F(x)) dx.$$