Let X = failure time

Initial model: $X \sim \text{Exponential}\ (\theta = 4)$

pdf: $f_0(x) = \frac{1}{4} e^{-x/4}$

New model: $f(x) = \begin{cases} b & 0 \le x \le 3 \\ a \cdot f_0(x) = \frac{a}{4} e^{-x/4}, & x > 3 \end{cases}$

$f(x)$ is continuous $\Rightarrow$ $f(x)$ is continuous at 3

$\Rightarrow \lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x)$

Want: $Pr(X \le 3) = \int_0^3 f(x)\,dx$

$= \int_0^3 b \cdot dx = 3b$

$\lim\limits_{x \to 3^-} f(x) = b$, $\quad \lim\limits_{x \to 3^+} f(x) = \frac{a}{4} e^{-3/4}$

So $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) \Rightarrow b = \frac{a}{4} e^{-3/4}$ ①

Since $f(x)$ is a p.d.f. with support $[0,\infty)$, we have

$$\int_0^\infty f(x)\,dx = 1$$

$$\Rightarrow \int_0^3 b\,dx + \int_3^\infty \frac{a}{4} e^{-x/4}\,dx = 1$$

$$\Rightarrow 3b + a\cdot\int_3^\infty \frac{1}{4} e^{-x/4}\,dx = 1 \qquad (2)$$

$$\int_3^\infty \frac{1}{4} e^{-x/4}\,dx = -e^{-x/4}\Big]_0^\infty = 0-(-e^{-3/4}) = e^{-3/4}$$

$$(2) \Rightarrow 3b + a\cdot e^{-3/4} = 1 \qquad (3)$$

Now we have two equations: $\begin{cases} b = \frac{a}{4} e^{-3/4} & (1) \\ 3b + a\cdot e^{-3/4} = 1 & (2) \end{cases}$

$$(1) \Rightarrow a = 4e^{3/4} b$$

$$(2) \Rightarrow 3b + 4e^{3/4} b\cdot e^{-3/4} = 1$$

$$\Rightarrow 3b + 4b = 1 \quad \Rightarrow b = \frac{1}{7}$$

$$\Rightarrow Pr(X \le 3) = 3b = \frac{3}{7} \approx 0.43.$$

Answer: A.