

## Question #5

Key: C

$$\pi(q | 1,1) \propto p(1 | q)p(1 | q)\pi(q) = 2q(1-q)2q(1-q)4q^3 \propto q^5(1-q)^2$$

$$\int_0^1 q^5(1-q)^2 dq = 1/168, \quad \pi(q | 1,1) = 168q^5(1-q)^2.$$

The expected number of claims in a year is  $E(X | q) = 2q$  and so the Bayesian estimate is

$$E(2q | 1,1) = \int_0^1 2q(168)q^5(1-q)^2 dq = 4/3.$$

The answer can be obtained without integrals by recognizing that the posterior distribution of  $q$  is beta with  $a = 6$  and  $b = 3$ . The posterior mean is

$$E(q | 1,1) = a/(a+b) = 6/9 = 2/3. \quad \text{The posterior mean of } 2q \text{ is then } 4/3.$$