Question #23
Key: E

Assume that $\theta > 5$. Then the expected counts for the three intervals are $15(2/\theta) = 30/\theta, 15(3/\theta) = 45/\theta,$ and $15(\theta - 5)/\theta = 15 - 75/\theta$ respectively. The quantity to minimize is

$$\frac{1}{5}\left[(30\theta^{-1} - 5)^2 + (45\theta^{-1} - 5)^2 + (15 - 75\theta^{-1} - 5)^2\right].$$

Differentiating (and ignoring the coefficient of 1/5) gives the equation $-2(30\theta^{-1} - 5)30\theta^{-2} - 2(45\theta^{-1} - 5)45\theta^{-2} + 2(10 - 75\theta^{-1})75\theta^{-2} = 0$. Multiplying through by θ^3 and dividing by 2 reduces the equation to $-(30 - 5\theta)30 - (45 - 5\theta)45 + (10\theta - 75)75 = -8550 + 1125\theta = 0$ for a solution of $\hat{\theta} = 8550/1125 = 7.6$.