

Question #29

Key: B

The probabilities are from a binomial distribution with 6 trials. Three successes were observed.

$$\Pr(3 | I) = \binom{6}{3} (.1)^3 (.9)^3 = .01458, \Pr(3 | II) = \binom{6}{3} (.2)^3 (.8)^3 = .08192,$$

$$\Pr(3 | III) = \binom{6}{3} (.4)^3 (.6)^3 = .27648$$

The probability of observing three successes is $.7(.01458) + .2(.08192) + .1(.27648) = .054238$. The three posterior probabilities are:

$$\Pr(I | 3) = \frac{.7(.01458)}{.054238} = .18817, \Pr(II | 3) = \frac{.2(.08192)}{.054238} = .30208, \Pr(III | 3) = \frac{.1(.27648)}{.054238} = .50975.$$

The posterior probability of a claim is then

$$.1(.18817) + .2(.30208) + .4(.50975) = .28313.$$