Question # 34
Answer: B

The likelihood is:

$$L = \prod_{j=1}^{n} \frac{r(r+1)\cdots(r+x_j-1)\beta^{x_j}}{x_j!(1+\beta)^{r+x_j}} \propto \prod_{j=1}^{n} \beta^{x_j}(1+\beta)^{-r-x_j}.$$

The loglikelihood is:

$$l = \sum_{j=1}^{n}\left[x_j \ln\beta - (r+x_j)\ln(1+\beta)\right]$$

$$l' = \sum_{j=1}^{n}\left[\frac{x_j}{\beta} - \frac{r+x_j}{1+\beta}\right] = 0$$

$$0 = \sum_{j=1}^{n}\left[x_j(1+\beta) - (r+x_j)\beta\right] = \sum_{j=1}^{n} x_j - rn\beta$$

$$0 = n\bar{x} - rn\beta; \quad \hat{\beta} = \bar{x}/r.$$