Question #58
Answer is B

Because the Bayes and Bühlmann results must be identical, this problem can be solved either way. For the Bühlmann approach, $\mu(\lambda) = v(\lambda) = \lambda$. Then, noting that the prior distribution is a gamma distribution with parameters 50 and 1/500, we have:

$$\mu = E(\lambda) = 50/500 = 0.1$$

$$v = E(\lambda) = 0.1$$

$$a = Var(\lambda) = 50/500^2 = 0.0002$$

$$k = v/a = 500$$

$$Z = 1500/(1500+500) = 0.75$$

$$\bar{X} = \frac{75+210}{600+900} = 0.19.$$

The credibility estimate is $0.75(0.19) + 0.25(0.1) = 0.1675$. For 1100 policies, the expected number of claims is 1100(0.1675) = 184.25.

For the Bayes approach, the posterior density is proportional to (because in a given year the number of claims has a Poisson distribution with parameter λ times the number of policies)

$\frac{e^{-600\lambda}(600\lambda)^{75}}{75!}\frac{e^{-900\lambda}(900\lambda)^{210}}{210!}\frac{(500\lambda)^{50}e^{-500\lambda}}{\lambda\Gamma(50)} \propto \lambda^{335}e^{-2000\lambda}$ which is a gamma density with parameters 335 and 1/2000. The expected number of claims per policy is 335/2000 = 0.1675 and the expected number of claims in the next year is 184.25.