Question #64
Answer is E

The model distribution is $f(x|\theta)=1/\theta, 0<x<\theta$. Then the posterior distribution is proportional to

$$\pi(\theta|400,600) \propto \frac{1}{\theta}\frac{1}{\theta}\frac{500}{\theta^2} \propto \theta^{-4}, \theta>600.$$

It is important to note the range. Being a product, the posterior density function is non-zero only when all three terms are non-zero. Because one of the observations was equal to 600, the value of the parameter must be greater than 600 in order for the density function at 600 to be positive. Or, by general reasoning, posterior probability can only be assigned to possible values. Having observed the value 600 we know that parameter values less than or equal to 600 are not possible.

The constant is obtained from $\int_{600}^{\infty} \theta^{-4} d\theta = \frac{1}{3(600)^3}$ and thus the exact posterior density is $\pi(\theta|400,600)=3(600)^3\theta^{-4}, \theta>600$. The posterior probability of an observation exceeding 550 is

$$\Pr(X_3>550|400,600)=\int_{600}^{\infty}\Pr(X_3>550|\theta)\pi(\theta|400,600)d\theta$$

$$=\int_{600}^{\infty}\frac{\theta-550}{\theta}3(600)^3\theta^{-4}d\theta=0.3125$$

where the first term in the integrand is the probability of exceeding 550 from the uniform distribution.