Question #69
Answer is B

For an exponential distribution the maximum likelihood estimate of the mean is the sample mean. We have

$$E(\bar{X}) = E(X) = \theta, Var(\bar{X}) = Var(X)/n = \theta^2/n.$$

$$cv = SD(\bar{X})/E(\bar{X}) = [\theta/\sqrt{n}]/\theta = 1/\sqrt{n} = 1/\sqrt{5} = 0.447.$$

If the above facts are not known, the loglikelihood function can be used:

$$L(\theta) = \theta^{-n}e^{-\Sigma x_j/\theta}, \quad l(\theta) = -n\ln\theta - n\bar{X}/\theta, \quad l'(\theta) = -n\theta^{-1} + n\bar{X}\theta^{-2} = 0 \Rightarrow \hat{\theta} = \bar{X}.$$

$$l''(\theta) = n\theta^{-2} - 2n\bar{X}\theta^{-3}, \quad I(\theta) = E[-n\theta^{-2} + 2n\bar{X}\theta^{-3}] = n\theta^{-2}.$$

Then, $Var(\hat{\theta}) = \theta^2/n.$