Question #75
Answer is D

$$E(X)=\int_{\delta}^{\infty}\frac{x}{\theta}e^{-(x-\delta)/\theta}dx=\int_{0}^{\infty}\frac{y+\delta}{\theta}e^{-y/\theta}dx=\theta+\delta$$

$$E(X^2)=\int_{\delta}^{\infty}\frac{x^2}{\theta}e^{-(x-\delta)/\theta}dx=\int_{0}^{\infty}\frac{y^2+2y\delta+\delta^2}{\theta}e^{-y/\theta}dx=2\theta^2+2\theta\delta+\delta^2.$$

Both derivations use the substitution $y=x-\delta$ and then recognize that the various integrals are requesting moments from an ordinary exponential distribution. The method of moments solves the two equations

$\theta+\delta=10$

$2\theta^2+2\theta\delta+\delta^2=130.6$

producing $\hat{\delta}=4.468.$

It is faster to do the problem if it is noted that $X=Y+\delta$ where *Y* has an ordinary exponential distribution. Then $E(X)=E(Y)+\delta=\theta+\delta$ and $Var(X)=Var(Y)=\theta^2$.