Question #76
Answer is D

The posterior density is proportional to the product of the probability of the observed value and the prior density. Thus, $\pi(\theta \mid N > 0) \propto \Pr(N > 0 \mid \theta)\pi(\theta) = (1 - e^{-\theta})\theta e^{-\theta}$.

The constant of proportionality is obtained from $\int_0^\infty \theta e^{-\theta} - \theta e^{-2\theta} d\theta = \frac{1}{1^2} - \frac{1}{2^2} = 0.75$.
The posterior density is $\pi(\theta \mid N > 0) = (4/3)(\theta e^{-\theta} - \theta e^{-2\theta})$.

Then,

$$\Pr(N_2 > 0 \mid N_1 > 0) = \int_0^\infty \Pr(N_2 > 0 \mid \theta)\pi(\theta \mid N_1 > 0) d\theta = \int_0^\infty (1 - e^{-\theta})(4/3)(\theta e^{-\theta} - \theta e^{-2\theta}) d\theta$$

$$= \frac{4}{3}\int_0^\infty \theta e^{-\theta} - 2\theta e^{-2\theta} + \theta e^{-3\theta} d\theta = \frac{4}{3}\left(\frac{1}{1^2} - \frac{2}{2^2} + \frac{1}{3^2}\right) = 0.8148.$$