Question # 92
Answer: C

Let N = number of prescriptions then $S = N \times 40$

n	$f_N(n)$	$F_N(n)$	$1-F_N(n)$
0	0.2000	0.2000	0.8000
1	0.1600	0.3600	0.6400
2	0.1280	0.4880	0.5120
3	0.1024	0.5904	0.4096

$$E(N) = 4 = \sum_{j=0}^{\infty}(1-F(j))$$

$$\begin{aligned} E\left[(S-80)_+\right] &= 40 \times E\left[(N-2)_+\right] = 40 \times \sum_{j=2}^{\infty}(1-F(j)) \\ &= 40 \times \left[\sum_{j=0}^{\infty}(1-F(j)) - \sum_{j=0}^{1}(1-F(j))\right] \\ &= 40(4-1.44) = 40 \times 2.56 = 102.40 \end{aligned}$$

$$\begin{aligned} E\left[(S-120)_+\right] &= 40 \times E\left[(N-3)_+\right] = 40 \times \sum_{j=3}^{\infty}(1-F(j)) \\ &= 40 \times \left[\sum_{j=0}^{\infty}(1-F(j)) - \sum_{j=0}^{2}(1-F(j))\right] \\ &= 40(4-1.952) = 40 \times 2.048 = 81.92 \end{aligned}$$

Since no values of S between 80 and 120 are possible,

$$E\left[(S-100)_+\right] = \frac{(120-100) \times E\left[(S-80)_+\right] + (100-80) \times E\left[(S-120)_+\right]}{120} = 92.16$$

Alternatively,

$$E\left[(S-100)_+\right] = \sum_{j=0}^{\infty}(40j-100) f_N(j) + 100 f_N(0) + 60 f_N(1) + 20 f_N(2)$$

(The correction terms are needed because $(40j-100)$ would be negative for j = 0, 1, 2; we need to add back the amount those terms would be negative)

$$\begin{aligned} &= 40\sum_{j=0}^{\infty} j \times f_N(j) - 100\sum_{j=0}^{\infty} f_N(j) + (100)(0.200) + (0.16)(60) + (0.128)(20) \\ &= 40\, E(N) - 100 + 20 + 9.6 + 2.56 \\ &= 160 - 67.84 = 92.16 \end{aligned}$$