Question #93
Answer: E

Method 1:

In each round,
N = result of first roll, to see how many dice you will roll
X = result of for one of the N dice you roll
S = sum of X for the N dice

$$E(X) = E(N) = 3.5$$
$$Var(X) = Var(N) = 2.9167$$

$$E(S) = E(N) * E(X) = 12.25$$
$$Var(S) = E(N)Var(X) + Var(N)E(X)^2$$
$$= (3.5)(2.9167) + (2.9167)(3.5)^2$$
$$= 45.938$$

Let $S_{1000} =$ the sum of the winnings after 1000 rounds

$$E(S_{1000}) = 1000 * 12.25 = 12{,}250$$
$$Stddev(S_{1000}) = sqrt(1000 * 45.938) = 214.33$$

After 1000 rounds, you have your initial 15,000, less payments of 12,500, plus winnings of S_{1000}.

Since actual possible outcomes are discrete, the solution tests for continuous outcomes greater than 15000-0.5. In this problem, that continuity correction has negligible impact.

$$\Pr(15000 - 12500 + S_{1000} > 14999.5) =$$
$$= \Pr((S_{1000} - 12250) / 214.33 > (14999.5 - 2500 - 12250) / 214.33) =$$
$$= 1 - \Phi(1.17) = 0.12$$

Method 2

Realize that you are going to determine N 1000 times and roll the sum of those 1000 N's dice, adding the numbers showing.

Let $N_{1000} =$ sum of those N's

$$E(N_{1000}) = 1000E(N) = (1000)(3.5) = 3500$$
$$Var(N_{1000}) = 1000Var(N) = 2916.7$$
$$E(S_{1000}) = E(N_{1000})E(X) = (3500)(3.5) = 12.250$$
$$Var(S_{1000}) = E(N_{1000})\,Var(X) + Var(N_{1000})\,E(X)^2$$
$$= (3500)(2.9167) + (2916.7)(3.5)^2 = 45.938$$

$$Stddev(S_{1000}) = 214.33$$

Now that you have the mean and standard deviation of S_{1000} (same values as method 1), use the normal approximation as shown with method 1.