

## Question #115

Key: D

Let  $X$  be the occurrence amount,  $Y = \max(X-100, 0)$  be the amount paid.

$$E[X] = 1,000$$

$$\text{Var}[X] = (1,000)^2$$

$$P(X > 100) = \exp(-100/1,000) = .904837$$

The distribution of  $Y$  given that  $X > 100$ , is also exponential with mean 1,000 (memoryless property).

So  $Y$  is  $\begin{cases} 0 \text{ with prob } .095163 \\ \text{exponential mean } 1000 \text{ with prob } .904837 \end{cases}$

$$E[Y] = .095163 \times 0 + .904837 \times 1,000 = 904.837$$

$$E[Y^2] = .095163 \times 0 + .904837 \times 2 \times (1,000)^2 = 1,809,675$$

$$\text{Var}[Y] = 1,809,675 - (904.837)^2 = 990,944$$

Alternatively, think of this as a compound distribution whose frequency is Bernoulli with  $p = .904837$ , and severity is exponential with mean 1,000.

$$\text{Var} = \text{Var}[N] \times E[X]^2 + \text{Var}[X] \times E[N] = p(1-p)(1,000,000) + p(1,000,000)$$