

Question #120

Key: E

$$X_{2002} = 1.05 \times X_{2001}$$

$$\begin{aligned} \text{so: } F\left(\frac{x_{2002}}{1.05}\right) &= 1 - \left[\frac{2000}{(x_{2002} / 1.05 + 2000)} \right]^2 \\ &= 1 - \left[\frac{2100}{x_{2002} + 2100} \right]^2 \end{aligned}$$

This is just another Pareto distribution with $\alpha = 2, \theta = 2100$.

$$E[X_{2002}] = 2100.$$

and

$$\begin{aligned} E[X_{2002} \wedge 3000] &= \left(\frac{2100}{1} \right) \times \left[1 - \left(\frac{2100}{(3000 + 2100)} \right) \right] \\ &= 2100 \times \left[\frac{3000}{5100} \right] = 1235 \end{aligned}$$

So the fraction of the losses expected to be covered by the reinsurance is $\frac{2100 - 1235}{2100} = 0.412$.

The total expected losses have increased to 10,500,000, so

$$C_{2002} = 1.1 \times 0.412 \times 10,500,000 = 4,758,600$$

$$\text{And } \frac{C_{2002}}{C_{2001}} = \frac{4,758,600}{4,400,000} = 1.08$$