

Question #125**Key: A**

Let N_1 , N_2 denote the random variable for # of claims for Type I and II in 2 years

X_1 , X_2 denote the claim amount for Type I and II

S_1 = total claim amount for type I in 2 years

S_2 = total claim amount for Type II at time in 2 years

$S = S_1 + S_2$ = total claim amount in 2 years

$\{S_1\} \rightarrow$ compound poisson $\lambda_1 = 2 \times 6 = 12$ $X_1 \sim U(0, 1)$

$\{S_2\} \rightarrow$ compound poisson $\lambda_2 = 2 \times 2 = 4$ $X_2 \sim U(0, 5)$

$$E(N_1) = \text{Var}(N_1) = 2 \times 6 = 12$$

$$E(S_1) = E(N_1)E(X_1) = (12)(0.5) = 6$$

$$\begin{aligned} \text{Var}(S_1) &= E(N_1)\text{Var}(X_1) + \text{Var}(N_1)(E(X_1))^2 \\ &= (12)\frac{(1-0)}{12} + (12)(0.5)^2 \\ &= 4 \end{aligned}$$

$$E(N_2) = \text{Var}(N_2) = 2 \times 2 = 4$$

With formulas corresponding to those for S_1 ,

$$E(S_2) = 4 \times \frac{5}{2} = 10$$

$$\text{Var}(S_2) = 4 \times \frac{(5-0)^2}{12} + 4\left(\frac{5}{2}\right)^2 = 33.\bar{3}$$

$$E(S) = E(S_1) + E(S_2) = 6 + 10 = 16$$

Since S_1 and S_2 are independent,

$$\text{Var}(S) = \text{Var}(S_1) + \text{Var}(S_2) = 4 + 33.\bar{3} = 37.\bar{3}$$

$$\Pr(S > 18) = \Pr\left(\frac{S-16}{\sqrt{39.\bar{3}}} > \frac{2}{\sqrt{37.\bar{3}}} = 0.327\right)$$

Using normal approximation

$$\begin{aligned} \Pr(S > 18) &= 1 - \Phi(0.327) \\ &= 0.37 \end{aligned}$$