Question #146
Key: D

Let α_j be the parameter for region *j*. The likelihood function is $L=\left(\prod_{i=1}^{n}\frac{\alpha_1}{x_i^{\alpha_1+1}}\right)\left(\prod_{i=1}^{m}\frac{\alpha_2}{y_i^{\alpha_2+1}}\right)$.

The expected values satisfy $\frac{\alpha_2}{\alpha_2-1}=1.5\frac{\alpha_1}{\alpha_1-1}$ and so $\alpha_2=\frac{3\alpha_1}{2+\alpha_1}$. Substituting this in the likelihood function and taking logs produces

$$l(\alpha_1)=\ln L(\alpha_1)=n\ln\alpha_1-(\alpha_1+1)\sum_{i=1}^{n}\ln x_i+m\ln\left(\frac{3\alpha_1}{2+\alpha_1}\right)-\frac{2+4\alpha_1}{2+\alpha_1}\sum_{i=1}^{m}\ln y_i$$

$$l'(\alpha_1)=\frac{n}{\alpha_1}-\sum_{i=1}^{n}\ln x_i+\frac{2m}{\alpha_1(2+\alpha_1)}-\frac{6}{(2+\alpha_1)^2}\sum_{i=1}^{m}\ln y_i=0.$$