Question #154
Key: E

For a compound Poisson distribution, *S*, the mean is $E(S|\lambda,\mu,\sigma)=\lambda E(X)=\lambda e^{\mu+0.5\sigma^2}$ and the variance is $Var(S|\lambda,\mu,\sigma)=\lambda E(X^2)=\lambda e^{2\mu+2\sigma^2}$. Then,

$$E(S)=E[E(S|\lambda,\mu,\sigma)]=\int_0^1\int_0^1\int_0^1 \lambda e^{\mu+0.5\sigma^2}2\sigma d\lambda d\mu d\sigma$$

$$=\int_0^1\int_0^1 e^{\mu+0.5\sigma^2}\sigma d\mu d\sigma=\int_0^1(e-1)e^{0.5\sigma^2}\sigma d\sigma$$

$$=(e-1)(e^{0.5}-1)=1.114686$$

$$v=E[Var(S|\lambda,\mu,\sigma)]=\int_0^1\int_0^1\int_0^1 \lambda e^{2\mu+2\sigma^2}2\sigma d\lambda d\mu d\sigma$$

$$=\int_0^1\int_0^1 e^{2\mu+2\sigma^2}\sigma d\mu d\sigma=\int_0^1 0.5(e^2-1)e^{2\sigma^2}\sigma d\sigma$$

$$=0.5(e^2-1)0.25(e^2-1)=0.125(e^2-1)^2=5.1025$$

$$a=Var[E(S|\lambda,\mu,\sigma)]=\int_0^1\int_0^1\int_0^1 \lambda^2 e^{2\mu+\sigma^2}2\sigma d\lambda d\mu d\sigma-E(S)^2$$

$$=\int_0^1\int_0^1\frac{2}{3}e^{2\mu+\sigma^2}\sigma d\mu d\sigma-E(S)^2=\int_0^1\frac{1}{3}(e^2-1)e^{\sigma^2}\sigma d\sigma-E(S)^2$$

$$=\frac{1}{3}(e^2-1)\frac{1}{2}(e-1)-E(S)^2=(e^2-1)(e-1)/6-E(S)^2=0.587175$$

$$k=\frac{5.1025}{0.587175}=8.69.$$