

Question #156

Key: C

There are $n/2$ observations of $N = 0$ (given $N = 0$ or 1) and $n/2$ observations of $N = 1$ (given $N = 0$ or 1). The likelihood function is

$$L = \left(\frac{e^{-\lambda}}{e^{-\lambda} + \lambda e^{-\lambda}} \right)^{n/2} \left(\frac{\lambda e^{-\lambda}}{e^{-\lambda} + \lambda e^{-\lambda}} \right)^{n/2} = \frac{\lambda^{n/2} e^{-n\lambda}}{(e^{-\lambda} + \lambda e^{-\lambda})^n} = \frac{\lambda^{n/2}}{(1 + \lambda)^n}. \quad \text{Taking logs, differentiating}$$

and solving provides the answer.

$$l = \ln L = (n/2) \ln \lambda - n \ln(1 + \lambda)$$

$$l' = \frac{n}{2\lambda} - \frac{n}{1 + \lambda} = 0$$

$$n(1 + \lambda) - n2\lambda = 0$$

$$1 - \lambda = 0, \quad \lambda = 1.$$