

Question #157

Key: D

The posterior density function is proportional to the product of the likelihood function and prior density. That is, $\pi(q | 1, 0) \propto f(1 | q)f(0 | q)\pi(q) \propto q(1 - q)q^3 = q^4 - q^5$. To get the exact posterior density, integrate this function over its range:

$$\int_{0.6}^{0.8} q^4 - q^5 dq = \left. \frac{q^5}{5} - \frac{q^6}{6} \right|_{0.6}^{0.8} = 0.014069 \text{ and so } \pi(q | 1, 0) = \frac{q^4 - q^5}{0.014069}. \text{ Then,}$$

$$\Pr(0.7 < q < 0.8 | 1, 0) = \int_{0.7}^{0.8} \frac{q^4 - q^5}{0.014069} dq = 0.5572.$$