Question #163
Key: D

Let S = score

$$E(S) = E(E(S|\theta)) = E(\theta) = 75$$

$$\begin{aligned} Var(S) &= E[Var(S|\theta)] + Var[E(S|\theta)] \\ &= E(8^2) + Var(\theta) \\ &= 64 + 6^2 \\ &= 100 \end{aligned}$$

S is normally distributed (a normal mixture of normal distributions with constant variance is normal; see Example 4.30 in Loss Models for the specific case, as we have here, with a normally distributed mean and constant variance)

$$\Pr ob[S < 90 | S > 65] = \frac{F(90) - F(65)}{1 - F(65)}$$

$$= \frac{\Phi\left(\frac{90-75}{10}\right) - \Phi\left(\frac{65-75}{10}\right)}{1 - \Phi\left(\frac{65-75}{10}\right)}$$

$$\frac{\Phi(1.5) - \Phi(-1.0)}{1 - \Phi(-1.0)} = \frac{0.9332 - (1 - 0.8413)}{1 - (1 - 0.8413)} = \frac{0.7745}{0.8413} = 0.9206$$

Note that (though this insight is unnecessary here), this is equivalent to per payment model with a franchise deductible of 65.