Question #166
Key: C

Write (i) as $\frac{p_k}{p_{k-1}} = c + \frac{c}{k}$

This is an (a, b, 0) distribution with $a = b = c$.

Which?

1. If Poisson, $a = 0$, so $c = 0$ and $b = 0$

   $p_1 = p_2 = ... = 0$

   $p_0 = 0.5$

   p_k's do not sum to 1. Impossible. Thus not Poisson

2. If Geometric, $b = 0$, so $c = 0$ and $a = 0$

   By same reasoning as #1, impossible, so not Geometric.

3. If binomial, a and b have opposite signs. But here $a = b$, so not binomial.

4. Thus negative binomial.

   $$1 = \frac{a}{b} = \frac{\beta/(1+\beta)}{(r-1)\beta/(1-\beta)} = \frac{1}{r-1}$$

   so $r = 2$

   $$p_0 = 0.5 = (1+\beta)^{-r} = (1+\beta)^{-2}$$

   $$1 + \beta = \sqrt{2} = 1.414$$

   $$\beta = \sqrt{2} - 1 = 0.414$$

   $$c = a = \beta/(1+\beta) = 0.29$$