Question #180
Key: A

From Question 9, $F(10) = 1 - \frac{10e^{-10/\theta}}{\theta} - e^{-10/\theta} = 1 - e^{-10/\theta}(1 + 10\theta^{-1}) = g(\theta)$.

$$g'(\theta) = -\frac{20}{\theta^2}e^{-20/\theta}(1 + 20\theta^{-1}) + e^{-20/\theta}\frac{20}{\theta^2} = -\frac{400e^{-20/\theta}}{\theta^3}.$$

At the maximum likelihood estimate of 6, $g'(6) = -0.066063$.

The maximum likelihood estimator is the sample mean. Its variance is the variance of one observation divided by the sample size. For the exponential distribution the variance is the square of the mean, so the estimated variance of the sample mean is 36/2 = 18. The answer is $(-0.066063)^2(18) = 0.079$.