Question #184
Key: D

The posterior distribution can be found from

$$\pi(\lambda \mid 10) \propto \frac{e^{-\lambda}\lambda^{10}}{10!}\left(\frac{0.4}{6}e^{-\lambda/6} + \frac{0.6}{12}e^{-\lambda/12}\right) \propto \lambda^{10}\left(0.8e^{-7\lambda/6} + 0.6e^{-13\lambda/12}\right).$$

The required constant is found from

$$\int_0^\infty \lambda^{10}\left(0.8e^{-7\lambda/6} + 0.6e^{-13\lambda/12}\right)d\lambda = 0.8(10!)(6/7)^{11} + 0.6(10!)(12/13)^{11} = 0.395536(10!).$$

The posterior mean is

$$E(\lambda \mid 10) = \frac{1}{0.395536(10!)}\int_0^\infty \lambda^{11}\left(0.8e^{-7\lambda/6} + 0.6e^{-13\lambda/12}\right)d\lambda$$

$$= \frac{0.8(11!)(6/7)^{12} + 0.6(11!)(12/13)^{12}}{0.395536(10!)} = 9.88.$$