

Question #187

Key: D

For the geometric distribution $\mu(\beta) = \beta$ and $v(\beta) = \beta(1 + \beta)$. The prior density is Pareto with parameters α and 1. Then,

$$\mu = E(\beta) = \frac{1}{\alpha - 1},$$

$$v = EVPV = E[\beta(1 + \beta)] = \frac{1}{\alpha - 1} + \frac{2}{(\alpha - 1)(\alpha - 2)} = \frac{\alpha}{(\alpha - 1)(\alpha - 2)},$$

$$a = VHM = Var(\beta) = \frac{2}{(\alpha - 1)(\alpha - 2)} - \frac{1}{(\alpha - 1)^2} = \frac{\alpha}{(\alpha - 1)^2(\alpha - 2)},$$

$$k = v/a = \alpha - 1, \quad Z = \frac{1}{1 + k} = \frac{1}{\alpha}.$$

The estimate is

$$\frac{1}{\alpha}x + \left(1 - \frac{1}{\alpha}\right)\frac{1}{\alpha - 1} = \frac{x + 1}{\alpha}.$$