Question #199
Key: E

The density function is $f(x) = \frac{0.2x^{-0.8}}{\theta^{0.2}} e^{-(x/\theta)^{0.2}}$. The likelihood function is

$$
\begin{aligned}
L(\theta) &= f(130) f(240) f(300) f(540)[1-F(1000)]^2 \\
&= \frac{0.2(130)^{-0.8}}{\theta^{0.2}} e^{-(130/\theta)^{0.2}} \frac{0.2(240)^{-0.8}}{\theta^{0.2}} e^{-(240/\theta)^{0.2}} \frac{0.2(300)^{-0.8}}{\theta^{0.2}} e^{-(300/\theta)^{0.2}} \frac{0.2(540)^{-0.8}}{\theta^{0.2}} e^{-(540/\theta)^{0.2}} e^{-(1000/\theta)^{0.2}} e^{-(1000/\theta)^{0.2}} \\
&\propto \theta^{-0.8} e^{-\theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2})}, \\
l(\theta) &= -0.8\ln(\theta) - \theta^{-0.2}(130^{0.2}+240^{0.2}+300^{0.2}+540^{0.2}+1000^{0.2}+1000^{0.2}) \\
&= -0.8\ln(\theta) - 20.2505\theta^{-0.2}, \\
l'(\theta) &= -0.8\theta^{-1} + 0.2(20.2505)\theta^{-1.2} = 0, \\
\theta^{-0.2} &= 0.197526, \quad \hat{\theta} = 3,325.67.
\end{aligned}
$$