Question #203
Key: C

For the geometric distribution, $\Pr(X_1 = 2 \mid \beta) = \dfrac{\beta^2}{(1+\beta)^3}$ and the expected value is β.

$$\Pr(\beta = 2 \mid X_1 = 2) = \frac{\Pr(X_1 = 2 \mid \beta = 2)\Pr(\beta = 2)}{\Pr(X_1 = 2 \mid \beta = 2)\Pr(\beta = 2) + \Pr(X_1 = 2 \mid \beta = 5)\Pr(\beta = 5)}$$

$$= \frac{\frac{4}{27}\frac{1}{3}}{\frac{4}{27}\frac{1}{3} + \frac{25}{216}\frac{2}{3}} = 0.39024.$$

The expected value is then 0.39024(2) + 0.60976(5) = 3.83.