Question #204
Key: D

The following derives the general formula for the statistic to be forgotten by time x. It would work fine, and the equations would look simpler, if you immediately plugged in $x=\frac{1}{2}$, the only value you want. Then the $x+\frac{1}{2}$ becomes 1.

Let *X* be the random variable for when the statistic is forgotten. Then $F_X(x|y)=1-e^{-xy}$

For the unconditional distribution of *X*, integrate with respect to y

$$
\begin{aligned}
F_X(x) &= \int_0^{\infty}\left(1-e^{-xy}\right)\frac{1}{\Gamma(2)y}\left(\frac{y}{2}\right)^2 e^{-y/2}dy \\
&= 1-\frac{1}{4}\int_0^{\infty} y\, e^{-y(x+1/2)}d\,y \\
&= 1-\frac{1}{4\left(x+1/2\right)^2}
\end{aligned}
$$

$$F(1/2)=1-\frac{1}{4\left(1/2+1/2\right)^2}=0.75$$

There are various ways to evaluate the integral in the second line:

1. Calculus, integration by parts
2. Recognize that $\int_0^{\infty} y\left(x+\frac{1}{2}\right)e^{-y\left(x+\frac{1}{2}\right)}dy$

   is the expected value of an exponential random variable with $\theta=\frac{1}{x+\frac{1}{2}}$
3. Recognize that $\Gamma(2)\left(x+\frac{1}{2}\right)^2 y e^{-y\left(x+\frac{1}{2}\right)}$ is the density function for a Gamma

random variable with $\alpha = 2$ and $\theta=\frac{1}{x+\frac{1}{2}}$, so it would integrate to 1.

(Approaches 2 and 3 would also work if you had plugged in $x=\frac{1}{2}$ at the start. The resulting θ becomes 1).