

Question #206**Key: B**

Frequency is geometric with $\beta = 2$, so

$$p_0 = 1/3, \quad p_1 = 2/9, \quad p_2 = 4/27$$

Convolutions of $f_X(x)$ needed are

| X | f | f^{*2} |
|-----|-----|----------|
| 5 | 0.2 | 0 |
| 10 | 0.3 | 0.04 |

$$\text{so } f_S(0) = 1/3, \quad f_S(5) = 2/9(0.2) = 0.044, \quad f_S(10) = 2/9(0.3) + 4/27(0.04) = 0.073$$

$$E(X) = (0.2)(5) + (0.3)(10) + (0.5)(20) = 14$$

$$E[S] = 2E(X) = 28$$

$$\begin{aligned} E[S - 15]_+ &= E[S] - 5(1 - F(0)) - 5(1 - F(5)) - 5(1 - F(10)) \\ &= 28 - 5(1 - 1/3) - 5(1 - 1/3 - 0.044) - 5(1 - 1/3 - 0.044 - 0.073) \\ &= 28 - 3.33 - 3.11 - 2.75 = 18.81 \end{aligned}$$

Alternatively,

$$\begin{aligned} E[S - 15]_+ &= E[S] - 15 + 15f_S(0) + 10f_S(5) + 5f_S(10) \\ &= 28 - 15 + (15)\left(\frac{1}{3}\right) + 10(0.044) + 5(0.073) \\ &= 18.81 \end{aligned}$$