

Question #209**Key: D**

For any deductible d and the given severity distribution

$$\begin{aligned} E(X - d)_+ &= E(X) - E(X \wedge d) \\ &= 3000 - 3000 \left(1 - \frac{3000}{3000 + d} \right) \\ &= (3000) \left(\frac{3000}{3000 + d} \right) \end{aligned}$$

$$\text{So } P_{2005} = (1.2)(3000) \left(\frac{3000}{3600} \right) = 3000$$

The following paragraph just clarifies the notation in the rest of the solution:

Let r denote the reinsurer's deductible relative to losses (not relative to reinsured claims). Thus if $r = 1000$ (we are about to solve for r), then on a loss of 4000, the insured collects $4000 - 600 = 3400$, the reinsurer pays $4000 - 1000 = 3000$, leaving the primary insurer paying 400.

Another way, exactly equivalent, to express that reinsurance is that the primary company pays the insured 3400. The reinsurer reimburses the primary company for its claims less a deductible of 400 applied to claims. So the reinsurer pays $3400 - 400 = 3000$, the same as before.

Expected reinsured claims in 2005

$$= (3000) \left(\frac{3000}{3000 + r} \right) = \frac{9,000,000}{3000 + r}$$

$$R_{2005} = (1.1) \left(\frac{9,000,000}{3000 + r} \right) = (0.55) P_{2005}$$

$$\begin{aligned} \frac{9,900,000}{3000 + r} &= (0.55)(3000) = 1650 \\ r &= 3000 \end{aligned}$$

In 2006, after 20% inflation, losses will have a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = (1.2)(3000) = 3600$.

The general formula for claims will be

$$E(X - d)_+ = (3600) \left(\frac{3600}{3600 + d} \right) = \frac{12,960,000}{3600 + d}$$

$$P_{2006} = 1.2 \left(\frac{12,960,000}{3000 + 600} \right) = 3703$$

$$R_{2006} = 1.1 \left(\frac{12,960,000}{3600 + 3000} \right) = 2160$$

$$R_{2006} / P_{2006} = 0.5833$$

[If you applied the reinsurer's deductible to the primary insurer's claims, you would solve that the deductible is 2400, and the answer to the problem is the same].