

Question #215

Key: A

$$E(X | \lambda) = \text{Var}(X | \lambda) = \lambda$$

$$\mu = v = E(\lambda) = \alpha\theta; a = \text{Var}(\lambda) = \alpha\theta^2; k = v/a = 1/\theta$$

$$Z = \frac{n}{n+1/\theta} = \frac{n\theta}{n\theta+1}$$

$$0.15 = \frac{\theta}{\theta+1}(1) + \frac{1}{\theta+1}\mu = \frac{\theta + \mu}{\theta+1}$$

$$0.20 = \frac{2\theta}{2\theta+1}(2) + \frac{1}{2\theta+1}\mu = \frac{4\theta + \mu}{2\theta+1}$$

From the first equation,

$$0.15\theta + 0.15 = \theta + \mu \text{ and so } \mu = 0.15 - 0.85\theta$$

Then the second equation becomes

$$0.4\theta + 0.2 = 4\theta + 0.15 - 0.85\theta$$

$$0.05 = 2.75\theta; \theta = 0.01818$$