

Question #218

Key: E

$$f(x) = -S'(x) = \frac{4x\theta^4}{(\theta^2 + x^2)^3}$$

$$L(\theta) = f(2)f(4)S(4) = \frac{4(2)\theta^4}{(\theta^2 + 2^2)^3} \frac{4(4)\theta^4}{(\theta^2 + 4^2)^3} \frac{\theta^4}{(\theta^2 + 4^2)^2} = \frac{128\theta^{12}}{(\theta^2 + 4)^3(\theta^2 + 16)^5}$$

$$l(\theta) = \ln 128 + 12 \ln \theta - 3 \ln(\theta^2 + 4) - 5 \ln(\theta^2 + 16)$$

$$l'(\theta) = \frac{12}{\theta} - \frac{6\theta}{\theta^2 + 4} - \frac{10\theta}{\theta^2 + 16} = 0; 12(\theta^4 + 20\theta^2 + 64) - 6(\theta^4 + 16\theta^2) - 10(\theta^4 + 4\theta^2) = 0$$

$$0 = -4\theta^4 + 104\theta^2 + 768 = \theta^4 - 26\theta^2 - 192$$

$$\theta^2 = \frac{26 \pm \sqrt{26^2 + 4(192)}}{2} = 32; \theta = 5.657$$