

## Question #219

Key: A

$$E(X | \theta) = \int_0^\theta x \frac{2x}{\theta^2} dx = \frac{2\theta}{3}; \text{Var}(X | \theta) = \int_0^\theta x^2 \frac{2x}{\theta^2} dx - \frac{4\theta^2}{9} = \frac{\theta^2}{2} - \frac{4\theta^2}{9} = \frac{\theta^2}{18}$$

$$\mu = (2/3)E(\theta) = (2/3) \int_0^1 4\theta^4 d\theta = 8/15$$

$$EVPV = v = (1/18)E(\theta^2) = (1/18) \int_0^1 4\theta^5 d\theta = 1/27$$

$$VHM = a = (2/3)^2 \text{Var}(\theta) = (4/9) \left[ 4/6 - (4/5)^2 \right] = 8/675$$

$$k = \frac{1/27}{8/675} = 25/8; Z = \frac{1}{1 + 25/8} = 8/33$$

Estimate is  $(8/33)(0.1) + (25/33)(8/15) = 0.428$ .