

Question #225

Key: C

The quantity of interest is $P = \Pr(X \leq 5000) = \Phi\left(\frac{\ln 5000 - \mu}{\sigma}\right)$. The point estimate is

$$\Phi\left(\frac{\ln 5000 - 6.84}{1.49}\right) = \Phi(1.125) = 0.87 .$$

For the delta method:

$$\frac{\partial P}{\partial \mu} = \frac{-\phi(1.125)}{1.49} = -0.1422; \quad \frac{\partial P}{\partial \sigma} = \frac{-1.125\phi(1.125)}{1.49} = -0.1600 \quad \text{where} \quad \phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2} .$$

Then the variance of \hat{P} is estimated as $(-0.1422)^2 0.0444 + (-0.16)^2 0.0222 = 0.001466$ and the lower limit is $P_L = 0.87 - 1.96\sqrt{0.001466} = 0.79496$.