

Question #226

Key: A

$$\begin{aligned}\Pr(\theta = 0.1 | X_1 = 1) &= \frac{\Pr(X_1 = 1 | \theta = 0.1) \Pr(\theta = 0.1)}{\Pr(X_1 = 1 | \theta = 0.1) \Pr(\theta = 0.1) + \Pr(X_1 = 1 | \theta = 0.3) \Pr(\theta = 0.3)} \\ &= \frac{0.1(0.8)}{0.1(0.8) + 0.3(0.2)} = \frac{4}{7}\end{aligned}$$

Then,

$$E(X_2 | \theta = 0.1) = 0(0.2) + 1(0.1) + 2(0.7) = 1.5$$

$$E(X_2 | \theta = 0.3) = 0(0.6) + 1(0.3) + 2(0.1) = 0.5$$

$$E(X_2 | X_1 = 1) = (1.5) \frac{4}{7} + (0.5) \frac{3}{7} = 1.071$$