Question #247
Key: D

Let *E* be the even of having 1 claim in the first four years. In four years, the total number of claims is Poisson(4λ).

$$\Pr(Type\, I \mid E) = \frac{\Pr(E \mid Type\, I)\Pr(Type\, I)}{\Pr(E)} = \frac{e^{-1}(0.05)}{\Pr(E)} = \frac{0.01839}{\Pr(E)} = 0.14427$$

$$\Pr(Type\, II \mid E) = \frac{e^{-2}(2)(0.2)}{\Pr(E)} = \frac{0.05413}{\Pr(E)} = 0.42465$$

$$\Pr(Type\, III \mid E) = \frac{e^{-4}(4)(0.75)}{\Pr(E)} = \frac{0.05495}{\Pr(E)} = 0.43108$$

$$Note: \Pr(E) = 0.01839 + .05413 + .05495 = 0.12747$$

The Bayesian estimate of the number of claims in Year 5 is:
0.14427(0.25) + 0.42465(0.5) + 0.43108(1) = 0.67947.