Question #254
Key: D

The posterior distribution is

$\pi(\lambda \mid data) \propto (e^{-\lambda})^{90}(\lambda e^{-\lambda})^{7}(\lambda^{2}e^{-\lambda})^{2}(\lambda^{3}e^{-\lambda})\frac{\lambda^{4}e^{-50\lambda}}{\lambda} = \lambda^{17}e^{-150\lambda}$ which is a gamma distribution with parameters 18 and 1/150. For one risk, the estimated value is the mean, 18/150. For 100 risks it is 100(18)/150 = 12.

Alternatively,

The prior distribution is gamma with $\alpha = 4$ and $\beta = 50$. The posterior will be continue to be gamma, with $\alpha' = \alpha$ + no. of claims = 4 + 14 = 18 and $\beta' = \beta$ + no. of exposures = 50 + 100 = 150. Mean of the posterior = α / β = 18/150 = 0.12.
Expected number of claims for the portfolio = 0.12 (100) = 12.