

Question #260

Key: E

$$\begin{aligned}\Pr(\theta = 8 | X_1 = 5) &= \frac{\Pr(X_1 = 5 | \theta = 8) \Pr(\theta = 8)}{\Pr(X_1 = 5 | \theta = 8) \Pr(\theta = 8) + \Pr(X_1 = 5 | \theta = 2) \Pr(\theta = 2)} \\ &= \frac{0.125e^{-5(0.125)}(0.8)}{0.125e^{-5(0.125)}(0.8) + 0.5e^{-5(0.5)}(0.2)} = 0.867035.\end{aligned}$$

Then,

$$E(X_2 | X_1 = 5) = E(\theta | X_1 = 5) = 0.867035(8) + 0.132965(2) = 7.202.$$