Question #267
Key: E

$$\Pr\left(\lambda=1 \mid X_1=r\right)=\frac{\Pr(X_1=r \mid \lambda=1)\Pr(\lambda=1)}{\Pr(X_1=r \mid \lambda=1)\Pr(\lambda=1)+\Pr(X_1=r \mid \lambda=3)\Pr(\lambda=3)}$$

$$=\frac{\frac{e^{-1}}{r!}(0.75)}{\frac{e^{-1}}{r!}(0.75)+\frac{e^{-3}3^r}{r!}(0.25)}=\frac{0.2759}{0.2759+0.1245(3^r)}.$$

Then,

$$2.98=\frac{0.2759}{0.2759+0.1245(3^r)}(1)+\frac{0.1245(3^r)}{0.2759+0.1245(3^r)}(3)$$

$$=\frac{0.2759+0.3735(3^r)}{0.2759+0.1245(3^r)}.$$

Rearrange to obtain

$$0.82218+0.037103(3^r)=0.2759+0.03735(3^r)$$

$$0.54628=0.00025(3^r)$$

$$r=7.$$

Because the risks are Poisson, (μ = EPV, a = VHM):

$$\mu=v=E(\lambda)=0.75(1)+0.25(3)=1.5$$

$$a=Var(\lambda)=0.75(1)+0.25(9)-2.25=0.75$$

$$Z=\frac{1}{1+1.5/0.75}=1/3$$

and the estimate is (1/3)(7) + (2/3)(1.5) = 3.33.