

## Question #269

Key: A

$$X \sim \text{Exp}(\theta)$$

$$\sum_{i=1}^n X_i \sim \Gamma(n, \theta)$$

$$\bar{X} \sim \Gamma(n, \theta/n)$$

$$E(\bar{X}^2) = (\theta/n)^2 (n)(n+1) = (n+1)\theta^2/n.$$

The second line follows because an exponential distribution is a gamma distribution with  $\alpha = 1$  and the sum of independent gamma random variables is gamma with the “ $\alpha$ ” parameters added. The third line follows because the gamma distribution is a scale distribution. Multiplying by  $1/n$  retains the gamma distribution with the “ $\theta$ ” parameter multiplied by  $1/n$ .