37. Solution: B

Duration is defined as $\frac{\sum_{t=1}^{\infty} t v^t R_t}{\sum_{t=1}^{\infty} v^t R_t}$, where for this problem v is calculated at i = 5% and R_t is D, the initial dividend amount, times $(1.02)^{t-1}$. Thus, the duration = $\frac{\sum_{t=1}^{\infty} t v^t D(1.02)^{t-1}}{\sum_{t=1}^{\infty} v^t D(1.02)^{t-1}} = \frac{\sum_{t=1}^{\infty} t v^t (1.02)^{t-1}}{\sum_{t=1}^{\infty} v^t (1.02)^{t-1}}$.

Using the mathematics of infinite geometric progressions (or just remembering the present value for a 1 unit geometrically increasing perpetuity immediate), the denominator = $v\frac{1}{(1-v(1.02))}$, which simplifies to $\frac{1}{i-.02}$. It can be shown* that the numerator simplifies to $\frac{1+i}{(i-.02)^2}$. So duration = numerator/denominator

$= \frac{1+i}{(i-.02)^2} / \frac{1}{i-.02} = \frac{1+i}{i-.02}$.

Thus, for i = .05, duration = (1.05)/.03 = 35.

Alternative solution:

A shorter alternative solution uses the fact that the definition of duration can be can be shown to be equivalent to – (1+i) P'(i)/P(i) where P(i) = $\sum_{t=1}^{\infty} v^t R_t$. Thus, in this case P(i) = $D\sum_{t=1}^{\infty} v^t (1.02)^{t-1} = D\frac{1}{i-.02}$ and

P'(i) (the derivative of P(i) with respect to i) = $D(-\frac{1}{(i-.02)^2})$. Thus, the duration = $-(1+i)\frac{-D(\frac{1}{(i-.02)^2})}{D\frac{1}{i-.02}}$ =

$\frac{1+i}{i-.02}$, yielding the same result as above.

----

*Note: The process for obtaining the value for the numerator using the mathematics of series simplification is:

Let S^{Num} denote the sum in the numerator.

Then $S^{Num} = 1\,v + 2\,(1.02)\,v^2 + 3\,(1.02)^2\,v^3 + \ldots + n\,(1.02)^{n-1}\,v^n + \ldots$ and (1.02)v $S^{Num} = 1\,(1.02)v^2 + 2\,(1.02)^2\,v^3 + \ldots + (n-1)\,(1.02)^{n-1}v^n + \ldots$

Thus, $(1-(1.02)v)\,S^{Num} = 1\,v + 1\,(1.02)v^2 + 1\,(1.02)^2\,v^3 + \ldots + 1\,(1.02)^{n-1}v^n + \ldots = v\frac{1}{(1-v(1.02))} = \frac{1}{(i-.02)}$

and $S^{Num} = \frac{1}{(i-.02)}/(1-(1.02)v) = \frac{1}{i-.02}/\frac{1+i-1.02}{1+i} = \frac{1}{i-.02}/\frac{i-.02}{1+i} = \frac{1+i}{(i-.02)^2}$.