

2. Solution: E

Accumulated value end of 40 years =

$$100 [(1+i)^4 + (1+i)^8 + \dots + (1+i)^{40}] = 100 ((1+i)^4)[1 - ((1+i)^4)^{10}] / [1 - (1+i)^4]$$

("Sum of finite geometric progression =

1<sup>st</sup> term times [1 - (common ratio) raised to the number of terms] divided by [1 - common ratio])"

and accumulated value end of 20 years =

$$100 [(1+i)^4 + (1+i)^8 + \dots + (1+i)^{20}] = 100 ((1+i)^4)[1 - ((1+i)^4)^5] / [1 - (1+i)^4]$$

But accumulated value end of 40 years = 5 times accumulated value end of 20 years

$$\text{Thus, } 100 ((1+i)^4)[1 - ((1+i)^4)^{10}] / [1 - (1+i)^4] = 5 \{ 100 ((1+i)^4)[1 - ((1+i)^4)^5] / [1 - (1+i)^4] \}$$

$$\text{Or, for } i > 0, 1 - ((1+i)^4)^{10} = 5 [1 - ((1+i)^4)^5] \text{ or } [1 - ((1+i)^4)^{10}] / [1 - ((1+i)^4)^5] = 5$$

But  $x^2 - y^2 = [x-y][x+y]$ , so  $[1 - ((1+i)^4)^{10}] / [1 - ((1+i)^4)^5] = [1 + ((1+i)^4)^5]$  Thus,  $[1 + ((1+i)^4)^5] = 5$  or  $(1+i)^4 = 4$ .

So X = Accumulated value at end of 40 years =  $100 ((1+i)^4)[1 - ((1+i)^4)^{10}] / [1 - (1+i)^4]$

$$= 100 (4^{1/5}) [1 - (4^{1/5})^{10}] / [1 - 4^{1/5}] = 6194.72$$

Alternate solution using annuity symbols: End of year 40, accumulated value =  $100(s_{\overline{40}|} / a_{\overline{4}|})$ , and end of year

20 accumulated value =  $100(s_{\overline{20}|} / a_{\overline{4}|})$ . Given the ratio of the values equals 5, then

$$5 = (s_{\overline{40}|} / s_{\overline{20}|}) = [(1+i)^{40} - 1] / [(1+i)^{20} - 1] = [(1+i)^{20} + 1]. \text{ Thus, } (1+i)^{20} = 4 \text{ and the accumulated value at the}$$

end of 40 years is  $100(s_{\overline{40}|} / a_{\overline{4}|}) = 100[(1+i)^{40} - 1] / [1 - (1+i)^{-4}] = 100[16 - 1] / [1 - 4^{-1/5}] = 6194.72$

Note: if  $i = 0$  the conditions of the question are not satisfied because then the accumulated value at the end of 40 years =  $40(100) = 4000$ , and the accumulated value at the end of 20 years =  $20(100) = 2000$  and thus accumulated value at the end of 40 years is not 5 times the accumulated value at the end of 20 years.