

6. Solution: C

$$\begin{aligned}\text{Cost of the perpetuity} &= v \cdot (Ia)_{\overline{n}|} + \frac{n \cdot v^{n+1}}{i} \\ &= v \cdot \left[\frac{\ddot{a}_{\overline{n}|} - nv^n}{i} \right] + \frac{n \cdot v^{n+1}}{i} \\ &= \frac{a_{\overline{n}|}}{i} - \frac{nv^{n+1}}{i} + \frac{nv^{n+1}}{i} \\ &= \frac{a_{\overline{n}|}}{i}\end{aligned}$$

Given $i = 10.5\%$,

$$\frac{a_{\overline{n}|}}{i} = \frac{a_{\overline{n}|}}{0.105} = 77.10 \Rightarrow a_{\overline{n}|} = 8.0955, \text{ at } 10.5\%$$

$$\therefore n = 19$$

Tips:

Helpful analysis tools for varying annuities: draw picture, identify "layers" of level payments, and add values of level layers.

In this question, first layer gives a value of $1/i$ (=PV of level perpetuity of 1 = sum of an infinite geometric progression with common ratio v , which reduces to $1/i$ at 1, or $v(1/i)$ at 0

2nd layer gives a value of $1/i$ at 2, or $v^2(1/i)$ at 0

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n^{th} layer gives a value of $1/i$ at n , or $v^n(1/i)$ at 0

Thus $77.1 = \text{PV} = (1/i)(v + v^2 + \dots + v^n) = (1/.105) a_{\overline{n}|.105}$

n can be easily solved for using BA II Plus or BA 35 Solar calculator