

Solution #33

Given: bond par value = 1000
term to maturity = 3 years
Coupon rate = 6% annual
interest rate

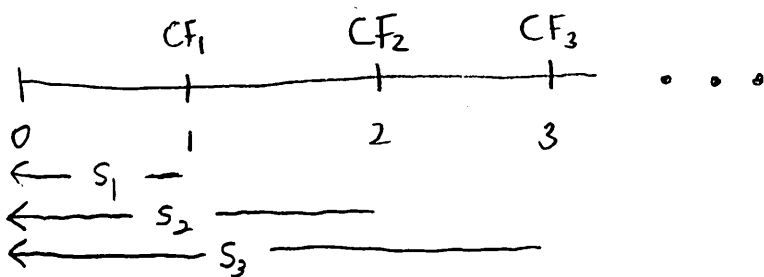
Goal: ① To calculate the price of bond at time 0 using spot rates.

② Calculate the price using forward rates.

Spot rates

The t -year spot rate s_t is the annual interest rate that can be earned on an investment made now to be repaid with interest in t years.

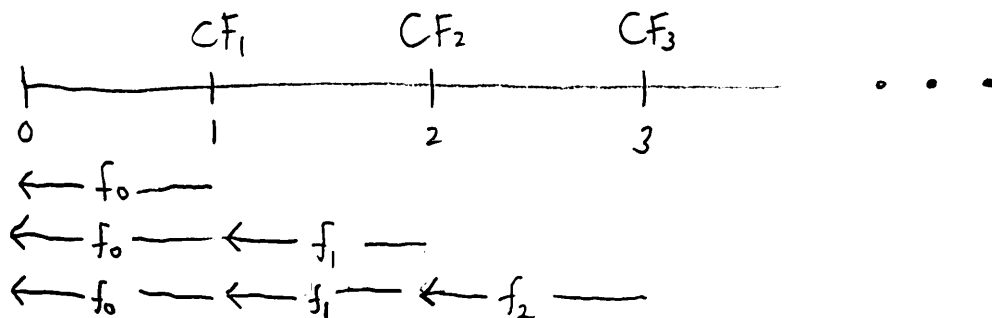
$$P_0 = \frac{CF_1}{(1+s_1)} + \frac{CF_2}{(1+s_2)^2} + \frac{CF_3}{(1+s_3)^3} + \dots$$



Forward rates

The forward rate beginning at time t years is the interest rate specified now for an investment beginning at time t and lasting until time $(t+1)$, f_t .

$$P_0 = \frac{CF_1}{(1+f_0)} + \frac{CF_2}{(1+f_0)(1+f_1)} + \frac{CF_3}{(1+f_0)(1+f_1)(1+f_2)} + \dots$$



Relationship: $(1+s_t)^t = (1+f_0)(1+f_1)\dots(1+f_{t-1})$

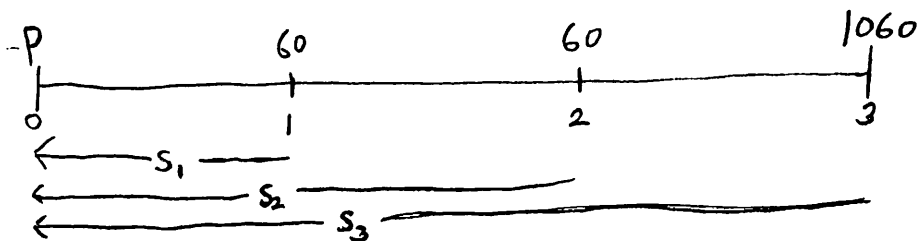
$$\Rightarrow f_{t-1} = \frac{(1+s_t)^t}{(1+s_{t-1})^{t-1}} - 1$$

Back to the problem:

$$s_1 = .07$$

$$s_2 = .08$$

$$s_3 = .09$$



$$P = \frac{60}{1.07} + \frac{60}{(1.08)^2} + \frac{1060}{(1.09)^3} = 926.03$$

$$f_0 = s_1 = .07$$

$$f_1 = (1+s_2)^2 = (1+f_0)(1+f_1)$$

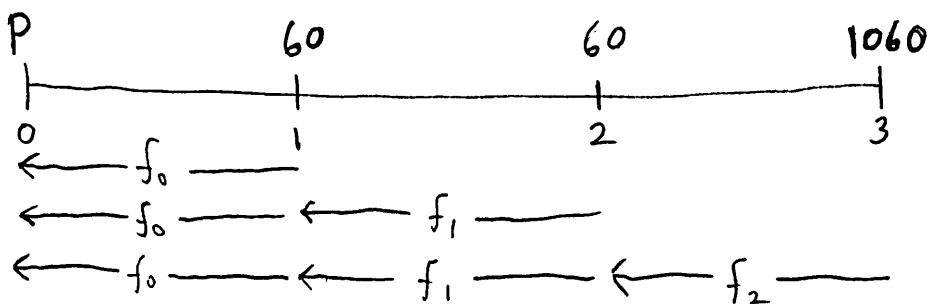
$$\Rightarrow (1.08)^2 = (1.07)(1+f_1)$$

$$f_1 = .090093458$$

$$f_2: (1+s_3)^3 = (1+f_0)(1+f_1)(1+f_2)$$

$$\Rightarrow (1.09)^3 = (1.07)(1.09009)(1+f_2)$$

$$f_2 = .110282157$$



$$P = \frac{60}{1.07} + \frac{60}{(1.07)(1.0900935)} + \frac{1060}{(1.07)(1.090093458)(1.110282157)}$$

$$= 926.03$$

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