FM #35

$$\text{Duration} = \frac{\sum_{t=1}^{n} t \cdot v^t \cdot C_t}{\sum_{t=1}^{n} v^t \cdot C_t}$$

$i = 8\%$

C_t = cash flow at time t.

$$\text{Price of bond} = \sum_{t=1}^{n} v^t \cdot C_t = 100 = \text{Denominator of Duration}$$

Derivative of Price (with respect to ytm)

$$= \frac{\partial}{\partial i}\left[\sum_{t=1}^{n} (1+i)^{-t} \cdot C_t\right] = -700$$

$$= -\sum_{t=1}^{n} t \cdot (1+i)^{-(t+1)} \cdot C_t$$

$$= -\sum_{t=1}^{n} t v^{t+1} \cdot C_t$$

$$-700 = -v \cdot \underbrace{\sum_{t=1}^{n} t v^t \cdot C_t}_{\text{Numerator of Duration}}$$

$$700(1 + \underset{\substack{\uparrow \\ 0.08}}{i}) = \text{Numerator of Duration} = 756$$

$$\text{So, Duration} = \frac{756}{700} = 7.56 \ \text{(C)}$$